Stats 2MB3, Tutorial 10

Mar 27th , 2015

Hypothesis Testing Procedures

- 1. Propose the null hypothesis H_0 and the alternative hypothesis H_a.
- 2. Determine a test statistics T, which is a function of the sample data on which the decision is to be based.
- 3. Select a rejection region which is the set of all test statistic values for which H_0 will be rejected.

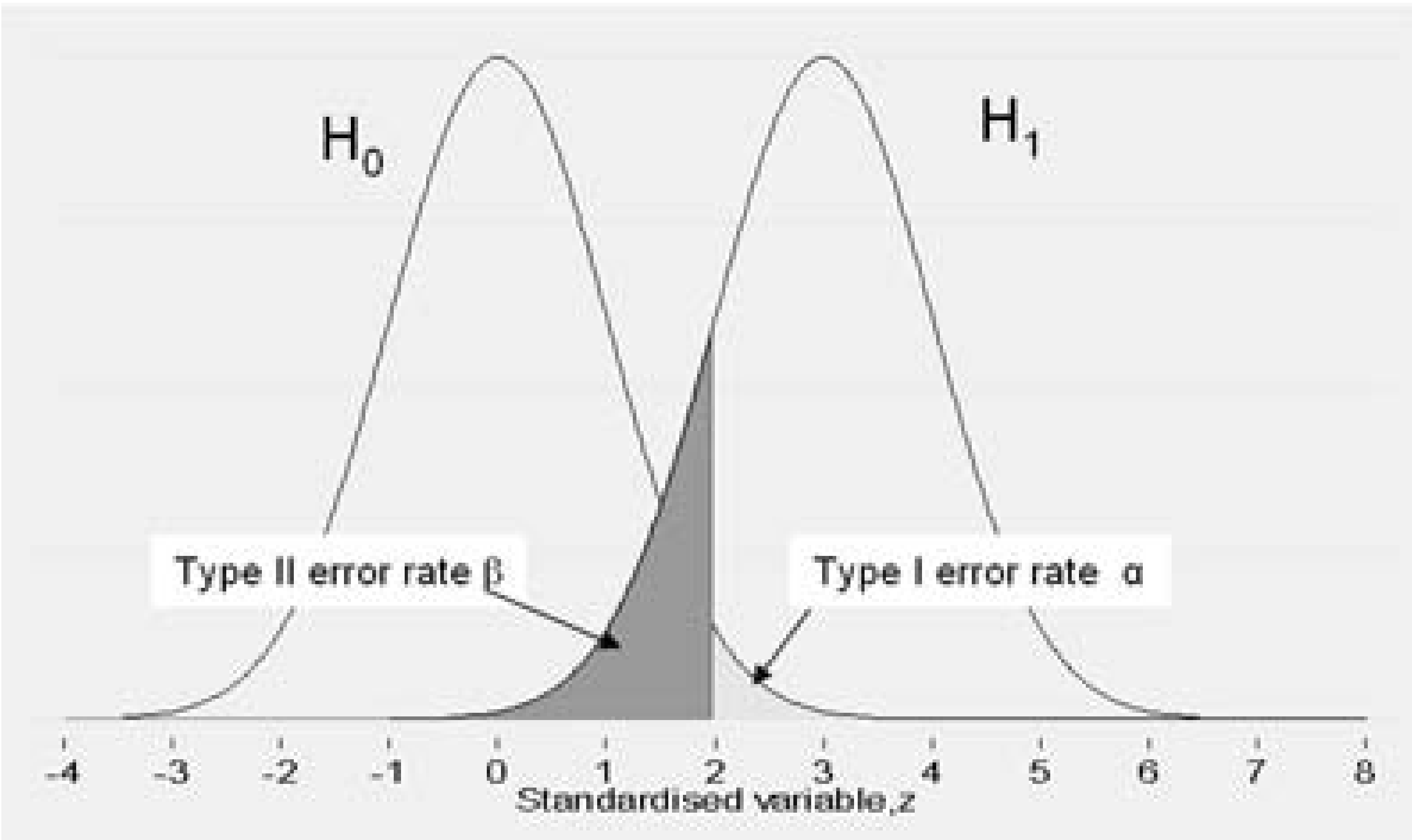
Type I & Type II errors

1. P(type I error)
   $=P(H_0$ is rejected when it is true)
   $=P(\text{sample data is at rejection region} \mid \theta \epsilon \theta_{H0})$
   $=\alpha$

Significance level is the probability of type I error.

2. P(type II error)
   $=P(H_0$ is not rejected when it is false)
   $=P(\text{sample data is not at rejection region} \mid \theta \epsilon \theta_{H1})$
   $=\beta(\theta)$

- We cannot reduce type I and type II errors at the same time.



- Let θ' denote a particular value of θ that exceeds the null hypothesis value θ_0.
- $\beta(\theta') = P(H_0$ is not rejected | $\theta = \theta')$

$$= P(\overline{X} < \theta_0 + z_\alpha \cdot \sigma / \sqrt{n} \mid \theta = \theta')$$

$$= P(\frac{\overline{X} - \theta'}{\sigma / \sqrt{n}} < z_\alpha + \frac{\theta_0 - \theta'}{\sigma / \sqrt{n}} \mid \theta = \theta')$$

$$= \Phi(z_\alpha + \frac{\theta_0 - \theta'}{\sigma / \sqrt{n}})$$

- As θ' increases, $\theta_0 - \theta'$ becomes more negative, so β(θ') will be small when θ' greatly exceeds θ_0.

Ex 7, page 309

- Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150 degree, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above 150, 50 water samples will be taken at randomly selected times and the temperature of each sample recorded. The resulting data will be used to test the hypotheses H_0 : μ=150 vs μ>150. In the context of this situation, describe type I and type II error. Which type of error would you consider more serious? Explain.

- Type I error is that the plant is not in compliance when in fact it is.
- Type II error is that the plant is in compliance when in fact it isn’t.
- Type II is more serious because our purpose is to “prohibit a mean discharge water temperature above 150”.

Ex 15, page 320

- Let the test statistic Z have a standard normal distribution when H_0 is true. Give the significance level for each of the following situation:
- a. H_a : $\mu > \mu_0$, rejection region $z \geq 1.88$;
- b. H_a : $\mu < \mu_0$, rejection region $z \leq -2.75$;
- c. H_a : $\mu \neq \mu_0$, rejection region $z \geq 2.88$ or $z \leq -2.88$.

- a) $\alpha=P(z \geq 1.88 \mid z$ has standard normal distribution$) = 1-\Phi(1.88)=0.0301$;
- b) $\alpha=P(z \leq -2.75 \mid z$ has standard normal distribution$) = \Phi(-2.75)=0.0300$
- c) $\alpha=P(z \leq -2.88$ and $z \geq 2.88 \mid z$ has standard normal distribution$)$

$$= \Phi(-2.88)+(1- \Phi(2.88))=0.004$$

Ex 17, page 321

- H_0 : μ=30,000 vs μ>30,000 based on a sample size n=16 from a normal population distribution with σ=1500.
- a) If the average value x_bar is 30960 and the level α is 0.01;
- b) For the level 0.01 test, what is β(30500)?
- c) For the level 0.01 test, if β(30500)=0.05, what sample size n is necessary?
- d) If the average x_bar=30960, what is the smallest α at which H_0 can be rejected.

- a) $$z = \frac{30960 - 30000}{1500/\sqrt{16}} = 2.56 > z_{\alpha} = 2.33$$

  so we reject H_0.

b) $$\beta(30500) = \Phi(2.33 + \frac{30000 - 30500}{1500/\sqrt{16}}) = \Phi(1.00) = 0.8413$$

c) When $\beta(30500) = \Phi(2.33 + \frac{30000 - 30500}{1500/\sqrt{n}}) = 0.05$

n= 143.

d) $$\alpha = 1 - \Phi(\frac{30960 - 30000}{1500/\sqrt{16}}) = 1 - \Phi(2.56) = 0.0052$$